Abstract Thought, Abstract Machine:

Alan Turing's Contemplation of Incompleteness

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Introduction

In 1936, as mathematicians strove to prove that math was a complete, non-contradictory system, Alan Turing wrote a paper that indicated otherwise.[1] In order to achieve his proof, he invented an unusual abstract machine. The universal machine, as he called it, was expressly built to demonstrate its own limit through a certain computational process; Turing built it in such a way that it would always fail to achieve the goal of deciding which problems in math were solvable. In its failure to achieve this goal, it essentially showed the inability of a mathematical method to exhaustively provide solutions.[2] The early theorem in which the machine first appeared continued the work begun by Kurt Gödel in the infamous Incompleteness theorems[3]; their collective research essentially showed that logical systems are never able to fully account for themselves, leaving gaps in their rational structure. This conclusion dealt a significant blow to certain sectors of the mathematics community at the time[4]—it clearly showed that math was fundamentally incomplete, robbing so-called positivists of a powerful, all-encompassing vision of rational totality.

My current project enters Turing's machine via this mathematical concept of incompleteness, opening a space in his work by reading it as a text among other philosophical texts in the 20th century. In short, I undertake a comparative study between the abstract machine in its historical context and a system of poststructuralist concepts, authored by Deleuze, Foucault and Massumi, that address the recursive but fundamentally incomplete nature of meaning. This comparative analysis shows that Turing enacts a radical act of contemplation that functions as a macrocosm of his machine's technical accomplishment. Through his mathematical texts, Turing accomplishes for himself, as a thinking, sensing human being, the same feat as the machine: he reveals his own limit, demonstrating in the process the pleasurable yet dangerous work involved in approaching this limit— in advancing upon what Foucault calls *le dehors*.[5] Turing antagonizes

this immanent limit in order to execute a reflexive contemplation on the debilitating incompleteness that consecrates us as human thinkers; it is this act of creative, despairing contemplation that reveals itself in my comparative process.

To draw a conclusion so steeped in continental thought from a mathematical theorem is, of course, contentious. For any writer in the humanities today to stake out a scientific text as territory is to submit oneself to scrutiny as an "intellectual impostor."[6] To explicate a theorem comparatively is to go further: it is to desecrate science not only by getting it wrong, but by treating a scientific proof as an open system in communication with other systems, other disciplines. Rather than closing down the potential for productive results, however, this imminent charge of desecration enables the project to gain momentum as a critical act of opening. Cracking the seal on math's system of objective proofs echoes Turing's own opening of math as a closed rational system; by contextualizing the math in terms of the philosophers most extensively attacked by accusers like Sokal and Bricmont, considering a proof in light of "theories too vague to be tested empirically,"[7] my project's breach of the theorem produces a space of creative reading in which one can see the production of meaning as itself a process of opening. Thus by methodologically mimicking Turing's death blow to mathematical totality, I aim to produce, out of this heated context of the betrayal of scientific precision, a place for Turing within a history of continental thought, a place that enables a reconceptualization of his work as a despairing engagement with constitutive absence.

Beyond this venue of the subverted abuse of science, however, my cross-disciplinary project begs a position within the realm of critical theory. An immediate option presents itself in Lévi-Strauss' conceptualization of an invariant element that manifests itself in all disciplines.[8] If one accepts that all structures are expressions of the same collective unconscious, then science,

philosophy and literature all share a kind of progenitor, and therefore share certain basic elements. In this sense, then, I could conceptualize my project as an attempt to close what the structural anthropologist has called the gap between science and myth[9] by tapping into human truth from either end, and joining them in the middle through comparative analysis. As truth-tapper, then, I would avoid the opprobrium of impostor and slip instead into the role of decipherer and uniter of human codes, boiling them down to the common element of an incompleteness so constitutive that it debilitates.

But while citing membership to this reductive scene might seem at first glance to be warranted, to stake this claim would in fact be antithetical to my project. The idea that a basic pattern gets continually erected and consistently expressed and enforced through Gödel and Turing's theorems and Deleuze, Foucault and Massumi's philosophies betrays a central tenet of the project itself: that thought can continually break down existing structures and can itself betray both rationality and representation. Rather than claiming to have discovered invariant structure, my project posits that what gathers these thinkers into a historical trend is precisely that their work betrays structure, including language, by indicating its limit, by revealing how it fails. In other words, I locate and value the anti-structural tendency in their thought, which delineates and betrays the human limit.

To situate Gödel and Turing's texts in relation to these continental philosophers is therefore in both cases to desecrate; I actively choose to betray both scientific objectivity and invariant structure, but not without exercising a certain *care*, one modeled on the formulation of meaning formation that Deleuze articulates in *The Logic of Sense* (and which I cover in depth later on). Deleuze writes that the process by which one expresses the meaning of one's last sentence in the current one, thereby linking them, amounts to "taking care of the sense."[10] From

this image, I draw a link between taking care and making meaning— continuing the critical process of opening expressed above, I want to suggest reciprocity between the scholarly attention that my project demands, and the incomplete process of meaning that it reveals. Taking care in this context therefore means working as a scholar to nurture the elements of the texts in question that continue to produce, to create, to unfold, to question themselves. In this sense enacting Deleuze's care means to take a profoundly anti-totalistic position, negating any coming to rest of the argument in a final goal, thereby affirming Turing's revealed antagonism of the limit by echoing it.

Abstract Thought, Abstract Machine

In order to prove that mathematics cannot be exhausted by a finite set of procedures, Alan Turing conceives, in 1936, of an abstract machine. He introduces the machine in "On Computable Numbers with an application to the *Entscheidungsproblem*," his first major mathematical paper.[11] In his invention of this unusual machine, Turing shows that no rational system can ever fully account for itself, producing contradictions and unknowns rather than settled questions. While his proof fundamentally rocks the world of mathematics, the work he undertakes has consequences that extend beyond the question of the completeness of a rational system, among them the problematization of meaning itself as a structural whole. Pushing this possibility into action, Turing enacts a despairing thought that approaches a contemplation of its own incompleteness. To understand this radical act of contemplation, he must be situated within a history of thinkers working against totality, because in thinking incompleteness on the levels of both rational machine and human thought, he refutes the idea that systems are defined by completeness, or that the unfolding of something is circumscribed by that something as goal. This constellation of thinkers includes Kurt Gödel, before Turing, with his Incompleteness Theorems[12]; Gilles Deleuze, with his explanation of how meaning gets made in *The Logic of Sense*[13]; and Michel Foucault, with his formulation of meaning's dissolution at the limit of thought, in "The Thought of the Outside."[14] Brian Massumi then ushers this tradition into the present by defining the limit of a human being as immanent to that being in *Parables for the Virtual*.[15] Massumi builds his theory with Deleuzeian and Foucauldian concepts, themselves evolved out of Turing's legacy of radical thought, which in turn is grounded in Gödel's theorem. Demonstrating these thinkers' relation to Turing's work on incompleteness will reveal the way in which systems of meaning are always torn between their own constitution and dissolution; this state of being torn will clarify, in turn, the precarious nature of Turing's thought.

To understand the inner workings of Turing's machine, one must first turn to Gödel's *On Formally Undecidable Propositions*, or his Incompleteness Theorems.[16] Gödel writes this text in 1931 to disprove the first two of three claims that David Hilbert made in 1928.[17] Hilbert's essentially positivist stance took math as a system in which every statement is verifiable—in other words, what makes math meaningful for him is that you can prove, one way or the other, whether something is true.[18] Hilbert called upon mathematicians to confirm that their rational system was consistently meaningful—that all its parts were fully provable. Thus his first claim was that mathematics is complete, or that every statement, for example, "1+1=2," can be either proved or disproved.[19] His second claim was that math is consistent, or that a statement is either true or false. Thus the statement "1+1=2" can never be true *and* false, but only one or the other; therefore math is free of contradiction.[20] If they could prove these two things, then math's meaning would be confirmed as practically total, an autonomous rational structure with seamless rules that expel contradiction and result in perfect, certain clarity.

Gödel's contribution to this scene is to prove otherwise. He manages to prove, for a particular calculus system, that unprovability is in fact the founding kernel of that system.[21] This rather ironic proof means both that there are always propositions in the system that can neither be proved nor disproved, and that something can be paradoxically true and not true at the same time. This impossibility means, for Gödel's system, that the system will always be both incomplete and inconsistent. While he only proves this for a single system, his method is so powerful that it stands for every formal system capable of representing arithmetic.[22]

Turing then introduces his theoretical machine in 1936 in order to disprove the third of Hilbert's contentions: that math is decidable, or that there is a definite method that can be applied to a statement to decide whether it is true or not. Since Gödel showed in his Incompleteness

Theorems that some statements cannot be proved either way, this claim, by the time Turing arrives on the scene, has been modified to ask whether there is a method to decide which statements are provable and which result in Gödelian paradox.[23] He begins by imagining a machine that provides a definite method to make conclusions about mathematical statements, basing it on what a human computer does when working out calculations. Accordingly, this machine is composed of three parts: the *executive unit* carries out operations such as reading and writing, the *control* ensures that instructions are correctly carried out, while the *store* is a store of information, and corresponds to a human computer's unlimited supply of paper, whether this is paper on which she does her calculations or that on which the book of rules that determine her behavior is printed. [24] This imaginary machine can therefore write, read, and erase symbols on a moving paper tape that he specifies should be of unlimited length. [25] Through this formulation, Turing writes the description for a "universal machine" that could theoretically complete any task, provided it could be fed appropriate directions.

To make the machine perform, Turing develops a method of coding these necessary directions for different tasks into special tables.[26] He uses a method of recursive definition to pack complex numbers into these simple instructive tables. R.B. Braithwaite provides a concise definition of this method of recursive construction:

> Recursive definition enables every number in a recursively defined infinite sequence to be constructed according to a rule, so that a remark about the infinite sequence can be constructed as a remark about the rule of construction and not about a given infinite totality.[27]

Here Braithwaite expresses how one can conceptualize a number in terms of its process, rather than its ultimate, potentially infinite place in the system that contains it. Consequently, he finds that he can finitely express any real number, whether finite or infinite, by making a table explaining the rule. For example, if he wants to express π, he creates a table defining the rule that produces the infinite decimal 3.14…. If he feeds the table to his machine, the machine will

produce the number theoretically, possibly taking up an unlimited amount of paper to do so (as, for example, in the case of an infinite decimal like π).[28] The machine effectively consists of these tables that Turing creates: it manifests as finite abstractions that express potentially infinite sequences.[29] Turing calls any number that he can create a table for a "computable number," because his machine can produce the number in its abstract space.[30] His machine is configured to produce every computable number, that is, every number for which one can write a table explaining its rule.[31] In this way, he creates a system that functions like a list of every single computable number in mathematics.

To show that there is no definite method to discover whether a given statement is provable, Turing applies a method invented by Georg Cantor to his computing machine.[32] Cantor originally used this method, called his Diagonal Proof, to prove that the set of all real numbers is greater than the set of all rational numbers.[33] The Diagonal Proof shows how no list of real numbers can contain all the real numbers, because any seemingly complete list reveals, through a certain diagonal method, yet another number not included in the list.[34] This paradox demonstrates how a seemingly rational system can produce irrationality, and how a system defined as complete can continue to generate additional members. When Turing applies this method to his machine, with its seemingly complete account of all computable numbers, he produces a similar result: despite the fact that the machine is defined as a complete account of all computable numbers, new numbers can always be generated through the diagonal method. Thus he shows that just as in Cantor the rational gives rise to the irrational, in his system the computable gives rise to the uncomputable—in other words, his computing machine can never exhaust itself, can never actually compute all the numbers it claims to account for; and any uncomputable number constitutes an unsolvable problem, because one will never discover, as Hilbert sought to learn,

whether it is provable or not. The fact that these unsolvable problems can always be made to appear crushes the idea that one could definitely separate out the decidable problems from the unsolvable ones. Thus, Turing concludes, there is no definite method to decide whether assertions are provable, since unsolvable problems will always reveal themselves. Therefore, mathematics cannot be exhausted by any finite procedure. Cantor, Gödel and Turing's results clearly indicate that a seemingly complete set or system will always reveal itself as incomplete (I use the term now in a more general sense to connote the disproof of all three of Hilbert's contentions as represented here, not only Gödel's technical result).

This proof situates Turing in a constellation of thinkers working against totality—against the idea that fundamental structures exist which are whole, complete and which can be fully accounted for. To subscribe to the notion of totality is to view a system as a kind of perfect sphere from which nothing ultimately escapes. Turing's relation to this concept of totality functions on several levels. His result in this early theorem clearly refutes the mathematical positivist conception of totality— it refutes, as we have seen, the completeness of a closed rational system, its ability to fully account for everything contained within it. Since positivism equates meaning with whether you can prove something true or false, then if ambiguity or contradiction results from an inquiry, the object in question is rendered meaningless. Put another way, positivists believe that if something is not clearly measurable, then it has no meaning. Therefore if humans, in their rational capacity, are unable to measure a given object, then that object in effect falls off the face of the mathematical earth and fails to enter into any meaningful relationship to that which has been rationally proved. This approach creates a logical totality dependent on human rationality's ability to decide "true" or "false" for any proposition—the truth of the positivist sphere is all that exists, and it exists completely. Turing, after Gödel,

clearly desecrates the totalistic positivist account when he shows that systems are fundamentally comprised of some elements that fail to be accounted for.

But while Gödel did start the ball rolling, and certainly showed that math lacks fundamentally what the positivists championed as the very definition of meaning, Gödel himself overturned the positivist conception of totality in favor of a different one. As a self-professed Platonist, he firmly believed, in opposition to the positivists, that there is an objective mathematical reality, a *Platonic* reality that we will never be able to fully grasp.[35] Thus for him meaning consists not in a thing's provability but in a correlation to mathematical Ideas. So while he contributed to the destruction of a positivist totality, its destruction for him cleared the way for a Platonic one—a perfect, complete sphere of math, parts of which we can discover but the whole of which we will never know.

Unlike Gödel, who professed openly his own Platonism, Turing only clarifies his position in this scheme through his math.[36] As we have seen, he further shows that there are clearly entities that form part of math that will continue to escape our grasp, but not necessarily because they form part of an ideal domain. My current project incorporates Gödel's results as part of the constellation it traces, but leaves his philosophical intention behind. The entities that exceed the system, for Turing, the computable numbers, do not *lack meaning* because they are contradictory or unsolvable, but neither do they indicate a perfect, unattainable total sphere; instead, Turing's results suggest a different nature for meaning, a nature other than provable or Platonic. Although Turing never stated so, viewing the situation through this project's ultimately comparative lens renders him both anti-positivist and anti-Platonic.

To flesh out the philosophical level on which Turing engages with totality, let us briefly situate the view, evolving out of Plato and Hegel, which Deleuze, Foucault and Massumi will

refute. In the realm of continental philosophy, positivism's expulsion of that contradictory stuff which cannot be clearly proven is replaced with a dialectical process whereby all contradiction is expelled and a total Truth is arrived at.[37] Unlike the positivist position, contradiction is not simply dismissed as meaningless but in fact fuels and forms the very movement of meaning as a struggle to resolve it. Therefore the Hegelian position essentially works out the contradiction that crops up in thought in order to approach a higher and higher level of truth. Hegel and Gödel share the general title of idealist; where Hegel's thinker approaches Absolute Truth as the digestion of all possible contradiction, Gödel approaches the Ideal sphere of mathematics, never to fully touch that perfection. But while Hegelian dialectics incorporates contradiction into its movement of meaning, that movement is nevertheless still a totalizing movement—it continues to define meaning in terms of an ultimate goal.

Enter Turing. His continually, newly revealed numbers, cropping up at the limit of his supposedly complete account, disprove on a mathematical level, and refute on a philosophical one, that one can define something in terms of its goal, in terms of its coming to rest in ultimate resolution. Turing's result literally renders this idea incoherent, and in the aftermath of this incoherence a new possibility grows. The move *away from* the conceptualization of a system (be it thought, a calculus, a language or a world) as a totality with a complete, definable set of elements allows for the existence of what Deleuze calls difference: something always differing, always else, always excluded.[38] In an anti-totalistic view, this conception of difference comes to occupy the general location that totality formerly did, but in a radical way; the new view posits proliferation and openness, rather than the certainty of a complete system, as the basis for meaning formation. In this vein, Turing's machine demonstrates that any system, or for this

project's purposes, any determinate thing, is always crucially in relation to indeterminacy rather than constituted as meaningful through its membership to a complete system.

In *The Logic of Sense*, Deleuze accounts for this constitutive indeterminacy, the result of withdrawal from the totalistic position, by elaborating the movement of what he calls "sense" as a subsistence that enables meaning to cohere.[39] Where Turing uses his machine to reveal the potential excess that haunts a system's meaning, Deleuze provides a philosophical definition of the excess itself. The key concepts we need to take from Deleuze are his definition of sense as a subsisting immanent limit, his differentiation of the corporeal and incorporeal, and his use of series to explain the proliferation of meaning. These three concepts will help to clarify how Turing not only reveals his machine as steeped in indeterminacy, but shows how his own position as a thinker engages incompleteness.

To provide a definition of sense, Deleuze distinguishes between the corporeal, which consists of bodies, words and things that exist, and an incorporeal realm of becoming, which subsists or inheres.[40] Sense belongs to the latter realm: it is an incorporeal entity that gives meaning to discourse.[41] Sense is a movement of difference: the differing of a thing from itself as well as what is not itself, differing from its own differing and always diverging from itself. This difference is the very movement of change: it consists of pure newness. In its differing, it enables the relation between words and concepts, traversing the gap between them and constituting their border.[42] Sense, Deleuze explains, is like an empty place that results from a constant inadequacy between the signifier and the signified[43]; thus systems of meaning are always incomplete because sense is partly absent, like a flickering phantom that enables the entire world, yet simultaneously renders it unfinished.[44]

Deleuze uses the idea of series to explain this cohering but 'un-completing' movement of sense.[45] He bases this idea on the mathematical concept of recursion—the same concept that Turing uses to express a potentially infinite number as a coded table expressing the number's rule construction, instead of its definition in a totality. To define it more generally than mathematically, recursion itself is the enabling of what comes next in a related string of numbers (or, to expand this view, of anything else, for example, words, colors or ideas). Recursion enables knowledge, experience or reality to continue: to snowball or to differ indefinitely, and it allows one to conceptualize this differing in terms of its "rule." In other words, like Turing's computable numbers, the movement of a series can be understood as an expression of its rule of construction; the logic of something can be understood in terms of the way it moves, as opposed to its destiny in a given telos, and in fact it *must* be understood this way, since after Turing, the series has no goal through which to define itself. Mathematically, this focus on movement as opposed to goal can be expressed as "K, K +1, K+2…," and can be translated as, "What happens? What happens next?"[46]

Accordingly, Deleuze describes sense as a self-displacement: as the entity that enables meaning to manifest, it is constantly moving in its role as incorporeal glue.[47] It links the variables in a series together, manifesting as the connective movement of the recursive. Deleuze explains,

> Sense is always presupposed as soon as I begin to speak: I would not be able to begin without this presupposition. In other words, I never state the sense of what I am saying. But… I can always take the sense of what I say as the object of another proposition whose sense, in turn, I cannot state.[48]

In other words, a series is like a chain: the beginning of each link is presupposed but unidentifiable until it is taken as the basis for the next link. This two-step process of first, the name that denotes something and second, the name that denotes the sense of the first name, is the minimal condition for the proliferation of meaning.[49] Making sense the object of the new

proposition amounts to what Deleuze calls "taking care of the sense"[50]—continuing the recursive movement into the next moment, into the next sentence. Therefore it is the care, and not any total outcome, that determines meaning.

Meaning's recursive movement is propelled by sense as it circulates, weaving signifiers and signifieds together, constituting their collective incompleteness. What is in excess in a given signifier is, Deleuze writes, an empty square, an always-moving place without an occupant. What is lacking, on the other hand, in the signified is an unknown occupant without a place--something always displaced. This excess and lack are two uneven sides of the same thing.[51] Systems of meaning are always incomplete because one side of sense is always absent from the other. It has a curious flip-flopping personality: its excess always refers to its own lack. This is how the meaningful world is constructed; it consists of ever-proliferating corporeal series, enabled by incorporeal, flip-flopping sense.[52]

Considering Turing's use of recursive definition through the lens of Deleuzeian sense clarifies his work in two linked ways: it philosophically focuses the nature of his machine's movement and, as will be covered later on, reveals a parallel between this movement and Turing's own dynamic as he contemplates difference. The theorem depends on recursion to show how mathematics fails to be exhausted by any finite procedure, to prove that any seemingly complete system will proliferate, producing more numbers that, while they are in excess of the system, are technically also part of the system. Their coming-into-existence simultaneously completes the apparently complete system, and extends the system further, just as Deleuze's second sentence completes the first, but manifests itself as incomplete, poised for the next recursive move as it anticipates its own unformed difference.

The paper tape on which Turing's machine inscribes its calculations and from which it derives its rules is the surface on which the machine's difference moves. While the strings of symbols inscribed on the tape are finite, they theoretically continue for infinity; while these symbols constitute a form of representation, they also demonstrate that representation's limit. A sensual transgression of the machine's own limit occurs on the tape's surface as the paper moves in and out of the machine's parts. While all three parts of the machine are essential for its functioning, the paper strip both holds the logic-as-movement for the machine itself and reveals its incompleteness as always recursive. While the strings of digits physically inscribed on the tape play out each computable number's logic-as-movement, the inscription of these digits leads to the thesis that none could be the final string, that none could constitute the completing factor of their system (in this case, the imagined complete set of computable numbers): that each one's role is to make way for the next surprise.

In standing as an expression of its own recursion, the machine points to the difference that is excluded in totality: the constant proliferation that meaning could not function without. One-sided and endless, the paper tape is the surface this difference fleetingly frequents, in between inscribed digits, hovering over each decision, each symbol marked, erased or passed over. The machine refers to the difference that enables, indeed forces, its system to remain open, to forever reveal new numbers: it refers to its own difference. Of course, in a way, anything, as actual and finite, both refers to its difference and expresses its incompleteness and transitoryness. In this sense, everything is a performance indexing both its own imminent disappearance and its recurrent change. Deleuze cites the interlacing of this disappearance and newness of change when he says that sense always refers to its own lack, flip-flopping between an empty space without an occupant and an occupant without a space to inhabit. Turing's machine differs from

any other given thing that carries with it its own incompleteness because the machine purposefully plays difference's drama out on its surface-- its mechanism points explicitly (as explicitly as is possible to point to something unlocatable) to the process through which meaning gets made. Thus the paper tape lights itself up as a surface of difference, like a looped, motion-activated security light.

While Deleuze's constructive formulation of meaning helps to show the machine's recursive movement, Foucault offers a contrasting theory of meaning's dissolution in "The Thought of the Outside." Like Deleuze's logic of sense, Foucault's theory evolves out of Gödel and Turing's refutation of totality, and it offers two important elements for our project: it further clarifies the abstract machine's self-revealing movement as approaching absence and impossibility, and it relates this movement explicitly to human thought. Gathering these two elements and adding them to the three we gained from Deleuze will enable us to make the jump from Turing's machine to the man himself: to shift focus from the machine's movement to the specifics of Turing's radical self-contemplation.

Foucault wants to set up a kind of thought that articulates its own end, one that can illuminate its dispersion at its own limit. He describes this, his thought's action, as "taking in only its invincible absence… in order to regain the space of its unfolding, the void serving as its site, the distance in which it is constituted and into which its immediate certainties slip the moment they are glimpsed."[53] In other words, he wants to practice a thought that immerses itself in its own absence and impossibility, writing, "What counts in men's thoughts is… the non-thought that systematizes them from the outset"[54] (here one sees the similarity to Deleuze's formulation of sense as that crucial non-entity which enables the corporeal realm). He calls this practice "the thought of the outside," describing it as using "language about the outside of

language, speech about the invisible side of words."[55] This discourse of the outside is "a listening less to what is articulated in language than to the void circulating between its words, to the murmur that is forever taking it apart."[56]

In the image of the outside as the void circulating between words, one sees clearly the link between Foucault's outside and Deleuze's formulation of sense as an always circulating, self-displacing empty place. But here also one finds a seeming difference between them: while Deleuze emphasizes the constitutive role that sense plays in its weaving together of words and things, Foucault instead emphasizes the process of meaning coming undone. By reading Deleuze's formulation of sense alongside Foucault's outside, one lays bare the mixed creative pleasure and solvent danger of meaning being made and unmade. Considering the contemporaneous constitution and dissolution of meaning, or the entering and falling of the pre-discursive into and out of discourse, reveals that the "next" of recursion is simultaneously a birth and death. Neither action could exist without the other, and they are in fact one and the same process: that of pure difference shifting. The constitution and dissolution of meaning occurs as difference breathes in and out or shifts in the night; this, difference's movement, manifests as meaning entering into and being destroyed at the limit of discourse.

Like a two-way mouth, the outside both offers up the abstract machine's strings of numbers and swallows them in a single movement (except that it is not single, but rather everywhere, although of course it is also nowhere). What the machine expresses is that the infinite (or the machine's difference from itself, that which (it) is not and can never be) serves as the un-centered core of its system's meaning. This is another way of saying, as Gödel did, that his system is dependent on incompleteness for its existence—it is a kind of "natural" state for the system. This, the machine's expression, is crucially related to Deleuze and Foucault's theories of

meaning because, in it, the machine acknowledges the outside as, to revisit the above quotation, "the void serving as its site." This realm of infinity is "the distance in which [the machine] is constituted…" and "into which its uncertainties slip the moment they are glimpsed…": what the machine renders certain (i.e. the computable numbers) immediately slips into the outside at the precise moment that it is rendered.

Foucault explains one manifestation of what he calls "the pure, most naked experience of the outside"[57] as a kind of attraction.[58] This attraction is, for Foucault, a contradictory voyaging toward the dissolving nothing that constitutes the outside. Foucault describes the dynamic of this attraction:

> The outside cannot offer itself as a positive presence… but only as an absence that pulls as far away from itself as possible, receding into the sign it makes to draw one toward it, as though it were possible to reach it. Attraction… has nothing to offer but the infinite void that opens beneath the feet of the person it attracts, the indifference that greets him as if he were not there, a silence too insistent to be resisted and too ambiguous to be deciphered and definitively interpreted…[59]

Thus attempting to follow or express the outside is a paradoxical experience. Turing's machine approaches that which can only offer itself "as an absence that pulls as far away from itself as possible," and when Foucault writes that this non-entity "[recedes] into the sign it makes to draw one toward it," we can see that even as the outside forms the machine's computable numbers, and even as we are drawn toward these signs, "as though it were possible to reach [them]," it dissolves into them and leaves us grasping the air, empty-handed. Foucault describes the attracting yet dissolving nature of a movement toward the outside as

> going toward the light in negligence of shadow, until it is discovered that the light itself is only negligence, a pure outside equivalent to a darkness that disperses, like a blown-out candle, the negligent zeal it has attracted.[60]

As well as standing in for the outside, Foucault's image of the dispersing blown-out candle represents the movement of recursion illuminated by Turing's machine. In this way, Turing's articulation of incompleteness constitutes a thought of the outside, embodied, as it were, in a

machine: the machine indicates the infinitely expanding space between digits, sequences and things, a nonexistent space constitutive of all meaning, all discourse. The expansive void of the outside, located, for Foucault, in between words, is the abyss Turing conceptually tightrope walks the edge of by thinking it in "On Computable Numbers."

Stopping at the observation that Turing, like Deleuze, formulates recursive movement as building the next link from the one before, would inhibit our consideration of Turing's contemplation, in that the consideration would be merely constructive, or unidirectional. In order to go further in our analysis, we must note carefully Foucault's emphasis of meaning's unraveling nature; recursion must be considered to encompass both constitution and dissolution, because meaning depends on the simultaneity of these processes to maintain itself, ever poised, flickering at its own limit. The notion that incompleteness lies at the core of any structure suggests that a given series of digits or, for example, images or musical notes is dependent on the yawning fact of its uncertainty for its meaning. The incompleteness both courses through a series as its links and surrounds that series as its limit; each aspect of this incompleteness constitutes the series in its very being. The outside is not simply outside, which is to say that it is not simply a gaping abyss that looms, waiting for one to die, outside the borders of being. It is precisely every being's immanent limit, the limit that we carry with us, which constitutes us as meaning beings. It is here, in Foucault's hinting at the outside's immanence to a system or being, that his debt to Gödel and Turing (and perhaps his similarity to Deleuze) is most evident. Thus in its grappling with incompleteness, Turing's thought of the outside teeters at this point of simultaneity where meaning comes together and falls apart.

Moving with Foucault from the domain of a meaning system like math into that of human thought itself, and specifically a reflexive thought that tries to contemplate its own limit, its own

dissolution, shifts our focus from the machine's flickering relation with meaning to Turing's own relation to the outside, as a thinker who intentionally approaches this limit. The theories outlined thus far have helped elucidate Turing's *machine's* position as simultaneously being constituted and dissolving at its own limit as strings of infinite numbers continue to reveal themselves. But to what degree can one apply this supposition regarding the movement of meaning to a thinking human being? The question now becomes: to what degree does Turing himself recur; to what degree is he coming apart? How is Turing himself always undone as he teeters at his own discursive limit? As we shall see, he not only expresses the machine's difference from itself, but in a kind of opening onto himself, he also references his own difference. What Turing finally accomplishes is a tweaking or antagonizing of his own immanent limit, marked by the simultaneity of constitution and dissolution, in order to reveal how meaning forms.

To consider the recursive movement of a thinking human being, and to explicitly describe the relation of this movement to a human's immanent limit, one requires a theory of embodied thought that qualifies as non-totalistic. Brian Massumi offers just this, a difference-focused problematization of the routinely attacked mind/body split, in his *Parables for the Virtual*. His theory of embodiment encompasses Foucault's notion of meaning's unraveling; carefully laying out Massumi's theory will help relate this unraveling motion to the thinker's own situation in meaning. Massumi builds his argument on all three of Deleuze's concepts so far outlined: the immanent limit of sense, the recursion of series and the corporeal/incorporeal duality. Deleuze assigns the minded body to the realm of the corporeal, while this minded body's ungraspable, cohering relation to itself belongs to the realm of incorporeality.[61] Massumi proceeds from this distinction by relating corporeality to abstraction in two ways. First, he radicalizes the relation by asking the question: What if the space of the body is really abstract?[62] What if the body is

inseparable from dimensions of lived abstractness, dimensions that it envelops yet which enable its existence? He proposes that the body's ultimate innards are as abstract, as incorporeal, as the insides of and space between atoms or neurons: the atomic and subatomic particles that make up "matter" are separated by voids larger than they themselves are, and each particle's own insides are un-solid, virtual, abstract.[63] The space in which atoms relate to each other is infinitely divisible; another way of saying this is that there are immeasurable gaps between things, the ungraspable nature of which gaps allows them to gain meaning as things.

This suggestion of constitutive immeasurability is akin to the paradoxical proposition of infinite divisibility advanced by Zeno in his Dichotomy: that (1) between two sides of the same street there are infinitely many points, (2) thus the street is uncrossable because one could never reach an infinity of successive points in a finite time, yet (3) one is able to cross the street.[64] This abstract space between the two sides of Zeno's street is a virtual space of potential that enables the finite street to cohere as such.[65] Thus the body, Massumi posits, is "the holding-together of… virtual innards as they fold out, recursive-durationally, in the loopy present, in determinate form and configuration, always provisional because always in becoming."[66] In other words, a minded body is a perpetual, always incomplete unfolding of itself. The abstract inside of the body is constantly becoming; it is constantly actualizing as graspable matter.

When Massumi uses the word "abstract" in this context, he is referring to habits, memories and tropisms: the recursive thought-perceptions that make up a person.[67] In this first sense, "abstract" means the relations that enable a person to cohere. The inside of the body does not consist only of intestines and bones, but also of the ungraspable relation of the body to itself. The body's recursion lies in its perpetual self-generation, which is enacted in its relation to itself. This self-relation is the "self-disjunctive coincidence of a thing's immediacy to its own

variation,"[68] or the continuity of sameness and heterogeneity (or, Heidegger would say, identity and difference).[69] To clarify this notion of "self-disjunctive coincidence," we need only to recall the abstract machine's contemporaneous inhabitation of its current state and its next recursive move. Consequently, again, the body is simultaneously itself and what it is becoming, as well as what it will have left behind. This is the body in series: its recursive self-differing movement from virtual to actual and back again.

In his second consideration of the body's relation to abstraction, Massumi formulates that there are two aspects to the body's self-conscious recursion: thought and perception are two poles of a single continuum. Perception, or feeling, is the anticipation of a next action in relation to the current and last one, while thought unfolds itself from feeling into concrete possibilities.[70] Perception senses potential, while thought is a systematic simplification of potential into possibility: thought turns anticipation into predictability. Abstraction is a separating out of thought from perception; the ability to abstract distinguishes human intelligence.[71] But abstraction paradoxically also intensifies perception, because "objectivity" makes more possibilities more anticipatable, thus more accessible as "nexts" or "mores." A perception's generalization or systematization returns to it as an augmentation of its singular multiplicity; its loss returns in the form of a gain.[72]

The inextricability of experience and extension into possibility ensures that every perception is also an analysis, and every concept that is grasped is also felt. Sensation is the name Massumi gives to a limit of experience that is immanent to every step of thought-perception along the continuum (his debt to Deleuze's formulation of sense as a linking subsistence is most evident here). [73] Sensation is the point of conversion where perception unfolds into thought and thought infolds into perception. It is "the registering of the multiplicity

of potential connections in the singularity of a connection actually under way" (again, one may recall the abstract machine, poised in anticipation of its next indication of incompleteness).[74] Thanks to this pure sense of multiplicity over singularity, the thought-perception continuum operates recursively: intelligence itself consists of the reality of an excess over the actual, which means that there is always a "next" or a "more." A given moment extends beyond itself in a way that is both thought and felt, anticipated, in the form of a yearning or tending.[75]

In the first sense of the word "abstract," then, Massumi means the virtual: Deleuze's incorporeal realm; Foucault's outside. Abstract refers to the infinitely divisible space between things, to a thing's relation to itself. When Massumi says that the inside of the body is really abstract, he doesn't mean that the center is abstract, but rather that there is neither center nor "complete" body; instead, the body in series unfolds *because of* the infinite non-substance that courses through it. This "coursing" doesn't actually exist: instead, like Gödel's incompleteness, Turing's undecidability, Deleuze's sense and Foucault's outside, it subsists, thus enabling what we think of, and live, as a minded body. In the second sense of the word "abstract," Massumi is talking about thought activity: conceptualization, generalization, and problematization. In the second sense, abstraction simplifies things, charting them out.

The first kind of abstraction, as pure virtuality, is the immanent limit of the second kind, as the extension of embodied thought. The incorporeality of sense can be thought of as pure meaning that follows thought's recursion, threading together perception and thought. But while it is always subsistent, it is also unlocatable: it never actualizes as itself, as pure sensation. Because of their perpetual folding into each other, thought is always sensual and feeling always conceptual; although thought both extends away from perception and links back to it, always passing through the intensity of sensation, thought can never actually access that enabling

sensation (notice here the similarity to Foucault's paradoxical attraction). Thought's role in the recursive "next" of intelligence is inseparable from embodied perception; sensation, as pure abstractness, is what binds thought and embodiment together as inseparable.

This embodied movement of pure abstractness further indicates that Foucault's outside is as much within us as outside us. The theoretical abyss that constitutes Foucault's outside is immanent to the self; it constitutes the self by constantly displacing itself, or as both Deleuze and Foucault put it, "circulating." While Turing's machine references the outside from which it enters into discourse and into which its digits plunge as they recur, his thought itself functions as a macrocosm of the machine, flickering at the point between the self and its limit, at that point of simultaneity where the outside violently stitches together and meticulously pulls apart meaning. Massumi's limit of pure sensation folds in and out as a single surface, constituting meaning as it moves, just as the outside constitutes and dissolves discourse in a continual movement of making things mean. This immanent outside is one's difference from oneself, the difference that Deleuze and Massumi have elaborated as the un-centered core of meaning.

From this un-centered core, Turing folds out into himself, into series, into thought. Massumi's supposition that intelligence is the reality of an excess over the actual further indicates that Turing's machine's movement echoes the nature of thought, in that the machine is constantly posed, in its singularity, at a junction of multiplicity. Turing's mathematical activities strain toward pure meaning (Massumi's sensation), but via the recursive movement so crucial to his proof, he simultaneously marks as evident the current, inevitable absence of this cohering entity. This unfolding of recursion is at the same time an "out-folding," or a coming apart. In this way, Turing and his machine are constantly disintegrating at their respective limits: their constitution as systems of meaning ensures this. Thus the idea of intelligence as recursion, as an

excess over the actual of any given moment, idea or image, illuminates the nature of Turing's thought to be a straining movement toward that abstractness which constitutes and courses through it, but can never be accessed.

Therefore if, following Massumi, Turing's insides are really abstract, then this straining that constitutes his most extensively abstract activities is also a highly intensive movement, in two ways. First, his thought activities in mathematics fuel the folding of new possibilities back into perception, which in turn feels out pending potential before delivering it back to thought to sort out. The thought extension that folds out from Turing's a-center runs seamlessly into and constitutes the abstract realm he studies, like a Möbius strip made from thought. Secondly, Turing strains, through his machine, toward the abstract realm of the outside, but because this realm is unlocatable, his is an a- or multi-directional straining. In his thought-extension, he touches upon the virtuality that makes the world have meaning, but simultaneously reaches for both his own virtual inners and their holding-together, the pure abstractness of his own self-relation.

This straining movement characterizes the suffering of a thought that approaches difference. Turing realizes that any system intended to catalog every element of that system will inevitably fail, because other elements will always be revealed. Therefore what Turing's theorem in fact expresses is the acute inadequacy of expression itself to account for a thing's logic. Expression's failure is precisely that it is an account: a formulated representation of movement-as-logic (the very idea of movement-as-logic is a static representation). The closest Turing's machine comes to expressing the suffering failure of thought is that it echoes the incompleteness of the world in its own inadequacy to grab hold of the enabling entity behind meaning's constitutive process. Even the expression of a thing's movement, which emphasizes its change, is

a cataloguing of that movement. In considering the machine's expression of its own inadequacy, one essentially witnesses expression shooting itself in the foot, which is the best it can do, under the circumstances (the best, at least, from the point of view of a philosophy approaching difference), because this self-mutilating act gives way to the refutation of totality. This is the closest one comes to being able to think difference: a weak echo of inherent incompleteness is, so far, the strongest philosophical tool. Turing's straining is akin to what James Bernauer calls Foucault's "cry of spirit"- his expression of a desire to inhabit the limit of thought in order to think difference.[76] What expression needs is a way to gallop alongside the proliferation of sense or the disintegration of the outside while retaining self-consciousness, but this is practically impossible, so one instead holds on for dear life to the (representational) idea of every thing and system's perpetual role as fragment. In this sense, the constellation of thinkers traced here graduates Hegel's "despairing thought" to a contemporary plane where, rather than grieve the constant loss of supposed ground as we approach totalizing Truth, we celebrate our inability to systematize and account for a whole as a sign that thought remains an open system. [77]

This, Turing's radical form of contemplation, constitutes, as do Gödel, Deleuze, Foucault and Massumi's theories, a pleasurable but dangerous kind of "playing with yourself": a toying with one's immanent limit in order to provoke oneself into going further. This "further" is a voyage of Foucauldian attraction into one's abstract depthlessness (rather than plunging into one's depth, it is a discovering of the most pure, unfolded patch of the outside's surface). It is in this topological sense that the question of thinking difference is a sensuous epistemological question: thinking is always sensuous to the extent that it is in constant contact with its own surface, the surface that ripples as discourse is constituted as such. In the current comparative context, the abstract machine's endless tape that indicates its own incompleteness clearly evokes

this notion of "its own surface." Further, the machine's structure echoes the thought extension that runs seamlessly from the a-center of Turing's body into the abstract realm he studies, forming a continual surface of thought. Explicitly trying to think difference simultaneously folds this surface further and straightens it out in order to see the outside: it is a flurried, multiplicitous movement in both directions at once, an attempted coming to rest in pure movement. Searching for one's own constitutive incompleteness can pull the thinking self apart: thought never quite reaches the pure orgasm of sensation, since in order to be thought it must either be about to pass or have just passed through the outside. For thought to reach its climax, for it to reach it and to rest there, would be to inhabit death. In this sense thought is always a tease: always promising a climax but fundamentally unable to follow through. Practicing a philosophy of difference is pleasurable precisely because it borders on danger: the danger of self-effacement, of (becoming) nothing. To think the outside is to connect to one's own incompleteness; in straining toward this "silence too insistent to be resisted and too ambiguous to be deciphered," one searches for one's own obliteration.

Thus Turing's machine can be read as a mathematical expression of the way in which the self is always torn between her own constitution and dissolution. By reading the theorem through Deleuze, Foucault and Massumi's theories of meaning, it becomes evident that it is precisely this torn-ness that makes meaning possible. The subject and action of Turing's radical contemplation is the way in which, as a system of meaning himself, he carries infinity with him, a barely perceptible, potentially debilitating, constitutive feature of his being. This means that as far as Turing can think, pure abstraction is just beyond him, enveloping him and bordering the furthest he can reach. His work in mathematics attempts to follow, and is followed at every turn by, its own incompleteness. This, finally, is Turing's mathematical thought: a thought inextricable from

feeling, folding out from and toward pure abstraction, in constant struggle with its own difference from itself.

[1] These mathematicians, most of them positivists, were responding, at this time, to Hilbert's program, which will be detailed in the essay itself. Turing responded directly to this program. Alan Turing, "On Computable Numbers, with an application to the *Entscheidungsproblem*" (Proc. London Mathematical Society: 2, 42, 1937), 230-265. http://www.abelard.org/turpap2/tp2-ie.asp.
[2] Turing, "Computable Numbers." He later expounds on its structure in Alan Turing, "Computing Machinery and Intelligence." (*Mind:* LIX, 236, 1950). http://www.abelard.org/turpap/turpap.htm.
[3] See Kurt Gödel, *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, trans. B. Meltzer (New York: Dover, 1992/1931).
[4] The most immediate example of this intellectual burglary is the shock and dismay exhibited by the Vienna Circle, detailed in, for example, Rebecca Goldstein, *Incompleteness* (New York: Atlas, 2005) 73.
[5] Michel Foucault, "The Thought of the Outside," in *Essential Works of Foucault 1954-1984, Volume 2: Aesthetics,* ed. James D. Faubion, essay trans. Brian Massumi (London: Penguin, 1998/1966), 147-169.
[6] As a young academic presenting work at various venues, my conference experience is inextricable from that of an immediate narrowing of the eyes, accompanied, in a condescending tone, by, "Have you read Sokal's experiment in *Social Text*?"
[7] Alan Sokal and Jean Bricmont, *Fashionable Nonsense* (New York: Picador, 1998/1997) 11.
[8] This idea is perhaps most readily available in Claude Lévi-Strauss, *Myth and Meaning* (New York: Schocken, 1979), for example, Part 1, "The Meeting of Myth and Science."
[99]Here, science and philosophy. Lévi-Strauss 6.
[10] Gilles Deleuze, *The Logic of Sense,* trans. Constantin V. Boundas (London: Continuum, 1990/1969) 31.
[11] Turing, "Computable Numbers."
[12] For one example of Gödel's refutation of positivist totality, see Gödel 32.
[13] Deleuze's *Logic* details this construction on multiple levels.
[14] Foucault 147-169.
[15] Brian Massumi, *Parables For the Virtual: Movement, Affect, Sensation* (Durham, North Carolina: Duke: 2002), 93.
[16] R. B. Braithwaite, "Introduction" to Gödel, 7.
[17] Andrew Hodges, *Alan Turing: The Enigma* (London: Vintage, 1992) 91.
[18] Goldstein 79.
[19] Hodges 91.
[20] David Foster Wallace, *Everything and More: A Compact History of Infinity* (New York: Norton: 2003), 282-85.
[21]"Introduction" to Gödel 32.
[22] "Introduction" to Gödel 7.
[23] Hodges 93.
[24] Turing, "Computing Machinery."
[25] Turing, "Computable Numbers."
[26] This universal machine becomes, of course, the model for the first British digital computer, build by Turing and others years later at the University of Manchester.
[27] "Introduction" to Gödel 12.

[28] Hodges 100.
[29] Ibid.
[30] Ibid.
[31] Ibid.
[32] Hodges 101.
[33] Foster Wallace 246-257.
[34] Hodges 101.
[35] One finds an example of his professed Platonism in his archived response to sociologist Burke Grandjean's inquiry into his position: "Dear Mr. Grandjean: Replying to your inquiries I would like to say first that I *don't* consider my work 'a facet of the intellectual atmosphere of the early 20th century' but rather the opposite." Goldstein 61. The early and socially painful contrast between Gödel's Platonism and his peers' positivism is simplistically dramatized in Janna Levin's recent *A Madman Dreams of Turing Machines*. Janna Levin, *A Madman Dreams of Turing Machines* (New York: Knopf, 2006).
[36] As far as I have been able to discover.
[37] For example, see his Introduction in G.W.F. Hegel, *Phenomenology of Spirit*, trans. A.V. Miller (Oxford: Oxford U, 1977) 46.
[38] To cite one of many examples, Deleuze 261.
[39] Deleuze 19.
[40] Deleuze 5.
[41] Deleuze 51.
[42] Deleuze 22.
[43] Deleuze 51.
[44] Deleuze 41.
[45] Deleuze 35.
[46] By contrast, a focus on goal would be expressed in a similar format as, "Where will this given object end up?"
[47] Deleuze 35.
[48] Deleuze 28.
[49] Deleuze 29.
[50] Deleuze 31.
[51] Deleuze 49-50.
[52] Deleuze's description of sense here as recursively flip-flopping absence marks one of his most obvious links to Derrida's conceptualization of *différance* as always differing-deferring, as always pushing off the signified into the next, as yet non-manifest sign. See, for example, Jacques Derrida, *Différance*, trans. by Alan Bass in *Critical Theory Since Plato* (Gainesville: U of Florida, 1986/1968) 123.
[53] Foucault 150.
[54] Quoted from an untranslated essay in James W. Bernauer, *Michel Foucault's Force of Flight: Toward An Ethics For Thought* (New York: Humanity Books: 1990), 56.
[55] Foucault 154.
[56] Ibid.
[57] Ibid.
[58] Foucault 151.
[59] Foucault 155.

[60] Foucault 157.
[61] Deleuze does not state this explicitly in *Logic*. However, through his elaboration of sense as an enabling relation, Deleuze implies that the system of the human body, like all other systems, gains meaning only through sense's incorporeal coherence. Therefore the human body's relation to itself subsists as the incorporeal.
[62] Massumi 177.
[63] Ibid.
[64] *Everything*, 47-59.
[65] This last sentence is not Zeno's precise reading but my reading of him. It should be noted that I am not attempting an Aristotelian reading of Zeno by writing the infinite off as merely possible or potential. Instead, I want to acknowledge this potential as a constitutive abstractness.
[66] Massumi 205.
[67] Ibid.
[68] Massumi 8.
[69] Martin Heidegger, *Identity and Difference* trans. Joan Staumbaugh (New York: Harper & Row: 1969), 17.
[70] Massumi 91.
[71] Massumi 92.
[72]Massumi 93.
[73] Massumi 92-3.
[74] Massumi 93.
[75] Massumi 92-3.
[76] Bernauer 182.
[77] Hegel 49.